Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

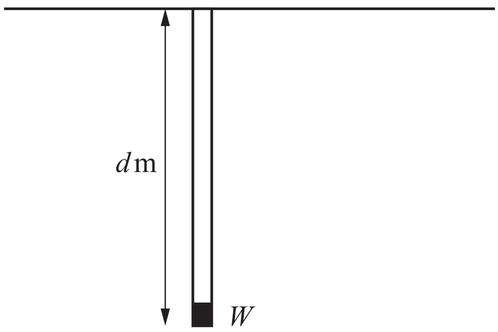
INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1



A small object W of weight 100 N is attached to one end of each of two parallel light elastic strings. One string is of natural length 0.4 m and has modulus of elasticity 20 N; the other string is of natural length 0.6 m and has modulus of elasticity 30 N. The upper ends of both strings are attached to a horizontal ceiling and W hangs in equilibrium at a distance d m below the ceiling (see diagram). Find d. **[5]**

2 A particle of mass 0.3 kg is projected horizontally under gravity with velocity 3.5 m s^{-1} from a point 0.4 m above a smooth horizontal plane. The particle first hits the plane at point A; it bounces and hits the plane a second time at point B. The distance AB is 1 m. Calculate

(i) the vertical component of the velocity of the particle when it arrives at A, and the time taken for the particle to travel from A to B, **[3]**

(ii) the coefficient of restitution between the particle and the plane, **[3]**

(iii) the impulse exerted by the plane on the particle at A. **[2]**

3 A particle P of mass 0.2 kg moves on a smooth horizontal plane. Initially it is projected with velocity 0.8 m s^{-1} from a fixed point O towards another fixed point A. At time t s after projection, P is x m from O and is moving with velocity v m s^{-1}, with the direction OA being positive. A force of $(1.5t - 1)$ N acts on P in the direction parallel to OA.

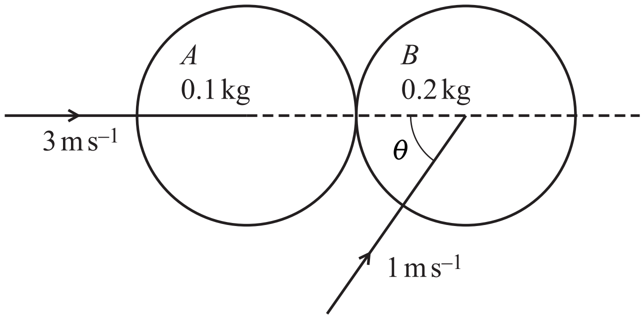
(i) Find an expression for v in terms of t. **[3]**

(ii) Find the time when the velocity of P is next 0.8 m s^{-1}. **[2]**

(iii) Find the times when P subsequently passes through O. **[4]**

(iv) Find the distance P travels in the third second of its motion. **[2]**

4 Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.1 kg and B has mass 0.2 kg. Immediately before the collision A is moving with speed $3\,\text{m}\,\text{s}^{-1}$ along the line of centres, and B is moving away from A with speed $1\,\text{m}\,\text{s}^{-1}$ at an acute angle θ to the line of centres, where $\cos\theta = 0.6$ (see diagram).

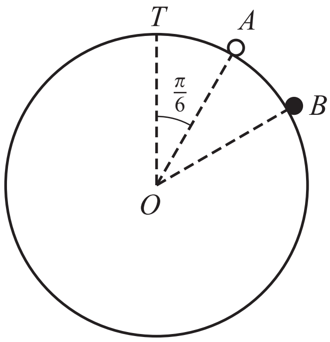


The coefficient of restitution between the spheres is 0.8. Find

(i) the velocity of A immediately after the collision, **[6]**

(ii) the angle turned through by the direction of motion of B as a result of the collision. **[5]**

5



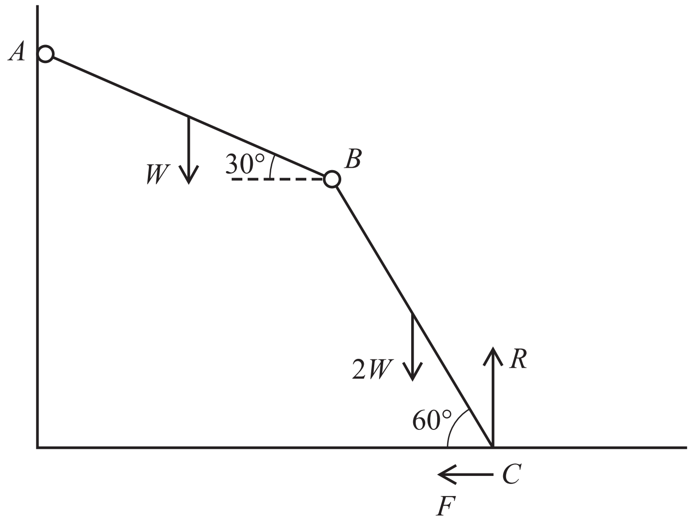
A fixed smooth sphere of radius 0.6 m has centre O and highest point T. A particle of mass m kg is released from rest at a point A on the sphere, such that angle TOA is $\frac{\pi}{6}$ radians. The particle leaves the surface of the sphere at B (see diagram).

(i) Show that $\cos TOB = \frac{\sqrt{3}}{3}$. **[6]**

(ii) Find the speed of the particle at B. **[2]**

(iii) Find the transverse acceleration of the particle at B. **[2]**

6 Two uniform rods AB and BC, each of length $2l$, are freely jointed at B. The weight of AB is W and the weight of BC is $2W$. The rods are in a vertical plane with A freely pivoted at a fixed point and C resting in equilibrium on a rough horizontal plane. The normal and frictional components of the force acting on BC at C are R and F respectively. The rod AB makes an angle 30° to the horizontal and the rod BC makes an angle 60° to the horizontal (see diagram).



(i) By considering the equilibrium of rod BC, show that $W + \sqrt{3}F = R$. **[2]**

(ii) By taking moments about A for the equilibrium of the whole system, find another equation involving W, F and R. **[4]**

(iii) Given that the friction at C is limiting, calculate the value of the coefficient of friction at C. **[5]**

7 A particle P of mass m kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity $39.2m$ N. The other end of the string is attached to a fixed point O. The particle is released from rest at O.

(i) Show that, while the string is in tension, the particle performs simple harmonic motion about a point 1 m below O. **[3]**

(ii) Show that when P is at its lowest point the extension of the string is 0.8 m. **[3]**

(iii) Find the time after its release that P first reaches its lowest point. **[6]**

(iv) Find the velocity of P 0.8 s after it is released from O. **[4]**

OCR
RECOGNISING ACHIEVEMENT

Question			Answer	Marks	Guidance	
1			Use of $T = \frac{\lambda e}{l}$	M1	Attempt at one tension; allow use of x	allow $2l$ for M1
				A1	$\frac{20(d-0.4)}{0.4}$ or $\frac{30(d-0.6)}{0.6}$	either term seen, accept in terms of x
			Weight = tension 1 + tension 2	M1		condone Wg and W/g
				A1	$100 = 50d - 20 + 50d - 30$	fractions and brackets removed
			($AW =$) 1.5 (m)	A1		
				[5]		
2	**(i)**		Use of correct formula Vert speed imm before bounce = 2.8 (m s^{-1}) Time between bounces = 0.286 (s) (2/7)	M1 A1 B1 **[3]**	$v^2 = 0^2 + 2 \times 9.8 \times 0.4$	or by energy
2	**(ii)**		Use of their t in a correct formula Vert speed imm after bounce = 1.4 (m s^{-1}) Coeff of rest = 0.5	M1 A1 B1ft **[3]**	$0 = u + 9.8 \times 0.5(t)$ Allow their value of t Their values for v after/v before	or $-u = u - 9.8t$ must be worked out to fraction or decimal; $0 \le e \le 1$
2	**(iii)**		Imp = change of mom $I = 1.26$ (N s)	M1 A1 **[2]**	$I = 0.3 \times (v) + 0.3 \times (u)$ Allow their u, v CAO	allow sign errors for M1, allow if answer implies use of their values
3	**(i)**		Use of $F = ma$	M1	$\frac{3}{2}t - 1 = 0.2\frac{dv}{dt}$	allow sign errors or m omitted
			Integrate correctly	A1	$v = \frac{15}{4}t^2 - 5t(+c)$	allow if c missing or wrong
			$v = \frac{15}{4}t^2 - 5t + 0.8$	A1		oe
				[3]		

Question			Answer	Marks	Guidance	
3	(ii)		Use vel = 0.8	M1	$\frac{15}{4}t^2 - 5t + 0.8 = 0.8$	ft their **(i)**
			t = 1.33 (s) or 1 1/3 (s)	A1	must come from correct equation for v	Accept 4/3
				[2]		
3	(iii)		Integrate to find x	M1*	At least 2 terms with powers increased by 1	
			$x = \frac{15}{12}t^3 - \frac{5}{2}t^2 + 0.8t$	A1	Need to state c = 0, or use limits	
			Solve for x = 0	*M1		
			t = 1.6 (s) or 0.4 (s)	A1	Both answers needed; must be from correct work to find equation	Ignore t = 0
				[4]		
3	(iv)		$x(3) - x(2)$	M1	Allow for $x(2)$ or $x(3)$ worked out from (iii)	13.65 or 1.6
			Distance is 12.05 (m)	A1		Accept 12 or 12.1
				[2]		
4	(i)		Conservation of momentum	*M1	Must have 4 terms	allow sign errors, $\cos\theta$ omitted
				A1	$0.1 \times 3 + 0.2 \times 1 \times \cos\theta = 0.1 \times a + 0.2 \times b$	a and b are vel components of A and B to right, respectively, after collision
			Newton's experimental law	*M1	Must have 4 terms and 0.8	allow sign errors, $\cos\theta$ omitted
				A1	$b - a = -0.8(1 \times \cos\theta - 3)$	
			Attempt to solve their 2 sim eqns	M1*	Dep both previous M marks	allow 1 slip
			0.12 in same direction as before	A1	Direction may be implied by working	withhold if direction stated to left
				[6]		
4	(ii)		b = 2.04	B1	Must be seen/used in (ii)	
			vel of B perp to line of centres = 0.8	B1	$(1 \times \sin\theta)$	
			Direction of B after collision makes angle	M1	$\tan\varphi = 0.8/2.04$;	Allow with their 0.8 and 2.04 (b from (i)); allow $\tan\varphi = 2.04/0.8$, if angle clear, leading to 68.4° for A1
			21.4° with line of centres	A1	or 0.374 rads	
			Angle turned through by B is 31.7°	A1ft	or 0.554 rads; allow +/−	$53.1(3) - \varphi$, $0.927 - 0.374$ rads
				[5]		

Question			Answer	Marks	Guidance	
5	(i)		Use of energy equation at A and B	M1	3 terms needed $mg0.6\cos\frac{\pi}{6} = mg0.6\cos\theta + \frac{1}{2}mv^2$	allow sign error, missing m / g / r
				A1		allow if θ replaced by $\varphi + \pi/6$
			$F = ma$ radially	M1 A1	$mg\cos\theta - R = \frac{mv^2}{0.6}$	allow sign error, missing m / g
			Use of $R = 0$	M1	May be incorporated in previous step	
			$\cos TOB = \frac{\sqrt{3}}{3}$ AG	A1 **[6]**	Completely correct	not given if decimals used for angle.
5	(ii)		Use of $\sqrt{3}/3$ in 'correct' equation in (i)	M1	$mg0.6\cos\frac{\pi}{6} = mg0.6\times\frac{\sqrt{3}}{3} + \frac{1}{2}mv^2$ or $mg\frac{\sqrt{3}}{3} = \frac{mv^2}{0.6}$	equation must have gained M1 in (i) but allow restart here
			1.84 (m s^{-1})	A1 **[2]**		
5	(iii)		Use of $F = ma$ tangentially	M1	$mg\sin\theta = ma$ seen	allow missing m/g, − sign; allow M1 if angular accel found
			8.00 (m s^{-2})	A1 **[2]**		
6	(i)		Moments about B for equilibrium of BC	M1	$2Wl\cos 60° + F2l\sin 60° = R2l\cos 60°$	3 moment terms, condone sin/cos errors and missing l. Need trig terms for M1
			$W + \sqrt{3}F = R$ AG	A1 **[2]**	Must be formula for R	correct, with sin/cos evaluated

Question			Answer	Marks	Guidance	
6	(ii)		Moments about A for equilibrium of whole system	M1	At least one of F and R terms must involve lengths of both rods	At least 3 moment terms, condone sin/cos errors, sign errors and $l/2l$ confusion/missing. Wrong use of forces at B gets M0
				A1	$Wl\cos 30 + 2W(2l\cos 30 + l\cos 60) + F(2l\sin 60 + 2l\sin 30) = R(2l\cos 30 + 2l\cos 60)$	4 terms, accept sin/cos errors and $l/2l$ confusion/missing and sign errors for A1
				A1	sin/cos left in, but correct	
			$W\left(\frac{5\sqrt{3}}{2}+1\right)+F\left(\sqrt{3}+1\right)=R\left(\sqrt{3}+1\right)$	A1	fully correct, oe. Mark final answer	accept $5.33W + 2.73F = 2.73R$, $W\left(\frac{13}{4}-\frac{3\sqrt{3}}{4}\right)+F=R$
					Allow full credit for candidates who work out internal forces at B and work correctly from there.	Eg $3R = \sqrt{3}F + 7.5W$
				[4]		
6	(iii)		Solving 2 sim equations to eliminate F or R	M1	Both equations must involve W, F and R	allow slips in working
				A1	$F = \frac{3\sqrt{3}}{4}W$	$F = 1.299\ W$
				A1	$R = \frac{13}{4}W$	$R = 3.25\ W$
			Use $F = \mu R$ to find μ	M1	At any point	
			$(\mu =)\frac{3\sqrt{3}}{13}$ (0.39970)	A1		Accept 0.4 if with correct working $5.33(R - 1.73F) + 2.73F = 2.73R$ $2.6R = 6.52F$
					Or eliminate W M1A1A1 Use $F = \mu R$ M1 cao A1	
				[5]		

Question			Answer	Marks	Guidance	
7	(i)		Use of $F = ma$ when string stretched	M1	Must have mg – tension term (involving 39.2m, 0.8 and x) = ma	allow if sign errors; x could be length or ext of string, or from eq^m pos.
				A1	$mg - \frac{39.2m(x-0.8)}{0.8} = m\ddot{x}$ $\ddot{x} = -49(x-1)$	$mg - \frac{39.2mx}{0.8} = m\ddot{x}$ leads to $\ddot{x} = -49(x-0.2)$ $mg - \frac{39.2(x+0.2)}{0.8} = m\ddot{x}$ leads to $\ddot{x} = -49x$
			Show $x = 1$ is centre of SHM or that $x = 1$ is equilibrium position.	B1 **[3]**	and state about $x = 1$	Convincingly
7	(ii)		By energy	M1	Must be PE term and EE term	Allow for missing '2', wrong 'g' or inconsistent lengths
				A1	$mg(0.8+e) = \frac{39.2me^2}{2\times0.8}$	Or $mgh = \frac{39.2m(h-0.8)^2}{2\times0.8}$ and $h = 0.8 + e$ $2.5e^2 - e - 0.8 = 0$
			$e = 0.8$ satisfies this equation **AG**	A1 **[3]**	Or by solving quadratic in e Allow full credit if done correctly from $v^2 = \omega^2(a^2 - x^2)$	Convincingly Allow integration of $v\frac{dv}{dx} = g - 49x$

Question			Answer	Marks	Guidance	
7	(iii)		For SHM, $\omega = 7$	B1		To be awarded if seen in **(i) or (iv)**
			$a = 0.6$	B1		or seen or used here
			Correct use of appropriate SHM distance equation	M1	$-0.2 = 0.6\cos(7t)$ or $-0.2 = 0.6\sin(7t)$	Allow +0.2, allow their a and ω
			$t = 0.272(9476)$ from bottom ($x = 1.6$) to $x = 0.8$	A1	Could be 0.0485 + 0.224	
			$t = 0.404(061)$ from O to $x = 0.8$	B1	Or $\frac{2\sqrt{2}}{7}$	May be seen first
			Time to reach lowest point = 0.677 s	A1ft	('0.273' + '0.404')	
				[6]		
7	(iv)		Use of $v = -a\omega\sin\omega t$ or $a\omega\cos\omega t$	M1	Must ft from their 'x' equation in (iii), or shown here	Allow use of their a and ω, sign error
			$v = -0.6 \times 7\sin 7t$	A1	or $0.6 \times 7\cos 7t$	
			Use of $t = 0.8 - 0.677 = 0.123$ after bottom point	B1ft	Or use of $t = 0.3475$ in 'cos' version	Must be between 0 and 0.8
			$v = 3.19$ (3.185677...)	A1	(−)3.187	Do not allow if direction stated to be down.
				[4]		